

hep-ph/0608126

August 2006

KU-TP005

# Constraining SuperWIMPy and Warm Subhalos with Future Submillilensing

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## Abstract

We propose to observe QSO-galaxy strong lens systems to give a new constraint on the damping scale of the initial fluctuations. We find that the future observation of submilliarc scale astrometric shifts of the multiple lensed images of QSOs would find  $\sim 10^{(3-9)} M_{\odot}$  subhalos inside the macrolens halo. The superweakly interacting massive particles (superWIMPs) produced from a WIMP decay and the warm dark matter (WDM) particles that predict a comoving damping scale larger than  $\sim 2$  kpc can be constrained if  $\sim 10^3 M_{\odot}$  subhalos are detected.

# 1 Introduction

There has been mounting evidence that most of the matter in the universe is not luminous but dark. Current observations such as the cosmic microwave background (CMB) suggest that the dark matter (DM) makes up about 25 % of the universe [1]. The dark matter is usually assumed to be a collisionless cold component (non-relativistic at the time of freeze-out), called as the cold dark matter (CDM). Weakly-interacting massive particles (WIMPs), such as the lightest neutralino in the supersymmetric standard model (SUSY SM) [2], and the lightest Kaluza-Klein particle in the universal extra dimension (UED) [3], are the popular CDM candidates. The predicted thermal relic abundances and the large-scale structure ( $\gtrsim 1$  Mpc) are in good agreement with the observed values.

However, the recent high-resolution  $N$ -body simulations on the CDM-based structure formation revealed various discrepancies on smaller scales ( $\lesssim 1$  Mpc). The first one is so-called the “missing satellite problem” [4]: the  $N$ -body simulations of the CDM particles predict significantly more virialized dark objects with mass  $M \lesssim 10^9 M_\odot$  (or subhalos) in the galaxy-sized halos with  $M \sim 10^{12} M_\odot$  than those observed around the Milky Way. The other one is called the “cusp problem” [5]: the CDM-based models also predict a cuspy profile for mass density distributions for the CDM halos [6] although the measurements of the rotation curves imply the presence of cores in the centers of the halo.

Although these discrepancies may be circumvented by some baryonic processes [7], it may be worthwhile to consider the other kinds of DM particles with different clustering properties. For instance, the superweakly interacting massive particles (superWIMPs) [8] or the warm dark matter (WDM) [9] particles can have large velocity dispersion at the epoch of radiation-matter equality. If the DM consists of the superWIMPs or the WDM particles, the number of less massive  $\lesssim 10^9 M_\odot$  subhalos is significantly reduced, and the cusp formation is also suppressed because the primordial fluctuations at the small scales ( $\lesssim 1$  Mpc) are damped.

In this note, we consider the possibilities of probing such subhalos with  $M \lesssim 10^9 M_\odot$  via strong lensing in which the image separations are on submilliarcsecond scales, called as “submillilensing”. Recently it has been pointed out that the future submillilensing observations of multiply-imaged QSO-galaxy lens systems can directly probe the mass scale of subhalos in the parent galaxy halo [10]. The submillilensing observation might resolve whether above problems originate from various baryonic contrivances or the nature of the DM particles. In section 2, we begin with the discussion of submillilensing and consider the possibility of detecting the small-mass subhalos via submillilensing in the next decade. In section 3, we study its implications to the superWIMP and the WDM scenarios. Section 4 is devoted to Conclusions and Discussion.

## 2 Submillilensing

In what follows, we discuss the possibilities of direct detection of subhalos with a mass of  $M \gtrsim 10^3 M_\odot$  via substructure lensing which is defined as lensing by  $M \lesssim 10^9 M_\odot$  subhalos

that perturb a “simple” strong lensing by  $\sim 10^{12}M_\odot$  parent galaxy halo. To date, about 10 quadruply-imaged gravitational lenses with flux ratio “anomalies” have been detected [11, 12, 13]. Here “anomaly” refers to an observed image flux ratio that does not agree with the ratio predicted by standard macrolens models with a smooth gravitational potential. From the radio and the mid-infrared observation, some of those lens systems are found to be consistent with the model in which the macrolens with a smooth potential is perturbed by subhalos [11, 12, 13, 14, 15]. Unfortunately, the mass scale of the subhalos has not been yet determined well.

First, we theoretically estimate the mass range of the subhalos that can perturb the fluxes of the multiple images produced by the parent galaxy halo with a mass of  $\sim 10^{12}M_\odot$ . In what follows, for simplicity, we assume that the low-mass subhalos are described by tidally-cut singular isothermal spheres (SISs) with a mass function  $dn/dm \propto m^{-2+\epsilon}$ , where  $0 < \epsilon \ll 1$ . At a distance  $r$  from the center, an SIS with a one-dimensional velocity dispersion  $\sigma_{\text{SIS}}$  has a density profile  $\rho(r) \propto \sigma_{\text{SIS}}^2/r^2$  [16]. If we further assume that the parent galaxy halo is also described by an SIS with a one-dimensional velocity dispersion  $\sigma$ , then the tidal radius  $r_t$  of the SIS subhalo with  $\sigma_{\text{SIS}}$  at a distance  $r$  from the center of the parent halo is approximately given by  $\sigma_{\text{SIS}}/r_t \approx \sigma/r$ , which yields the mass of the tidally-cut SIS as  $m \propto r\sigma_{\text{SIS}}^3$ . The effects of deviation from the assumption made here will be discussed later.

The lensing cross section by a subhalo is proportional to the square of the Einstein angular radius  $\theta_E$ , and the radius for the SIS is proportional to  $\sigma_{\text{SIS}}^2$  [16]. Using the equations for the mass function and the tidal radius, the substructure lensing cross section per logarithmic mass interval is given by

$$d\tau \propto \theta_E^2 dn \propto \sigma_{\text{SIS}}^4 dn \sim m^{1/3+\epsilon} d(\ln m) \quad (2.1)$$

where we have assumed that the distance  $r$  of the subhalo to the halo center is approximately equivalent to the Einstein radius of the parent halo  $r_E$ . This approximation is verified because the substructure lensing cross section depends weakly on the distance to the center  $r$  as  $\propto r^{-4/3}$ . The contribution from the distant halos in the line of sight can boost the cross section by a factor [17]. Thus, the contribution of massive subhalos is significant in the substructure lensing as long as the mass function satisfies  $\epsilon > -1/3$ . Assuming that the mass function with  $0 < \epsilon < 0.2$ , observed for the massive scales  $M \gtrsim 10^9 M_\odot$  [18], is applicable to the mass scale  $10^3 M_\odot \lesssim M \lesssim 10^9 M_\odot$ , the lensing probability by an intermediate-mass subhalo  $\sim 10^3 M_\odot$  is reduced by  $\sim 10^{-(3-4)}$  in comparison with the probability by a massive subhalo  $\sim 10^9 M_\odot$ .

If the subhalos have a different mass profile such as the NFW profile [6], the contribution of less massive subhalos can be further reduced. Because the ratio between the Einstein radius of the perturber and the tidal radius at a fixed distance  $r$  decreases as the subhalo mass decreases, the lensing cross section for a less massive subhalo is susceptible to the inner less cuspy profile. Thus, we expect that the contribution from  $M \lesssim 10^3 M_\odot$  subhalos is negligible in alternating the amplitude of the flux of multiple images.

Next, we explore the possibility of measuring the mass scale of these subhalos from astrometric shifts of the multiply lensed images. For SISs, the order of the magnitude of the astrometric shifts is typically given by the size of the angular Einstein radius  $\theta_E$  [19]. For an SIS with one-dimensional velocity dispersion  $\sigma_{\text{SIS}}$  at  $\sim \text{Gpc}$ ,  $\theta_E$  is approximately given by

$$\theta_E \sim 10 \left( \frac{\sigma_{\text{SIS}}}{20 \text{ km s}^{-1}} \right)^2 \text{ mas.} \quad (2.2)$$

Thus, observation with angular resolution of submilliarcsecond scales ( $\sim 0.01 \text{ mas}$ ), which will be achieved in the next generation satellite VLBI mission such as the VSOP2 [20], can reveal subhalos with one-dimensional velocity dispersion  $\sigma_{\text{SIS}} > 0.6 \text{ km s}^{-1}$ . It corresponds to  $M \gtrsim 10^3 M_\odot$  at the distance equal to the Einstein radius  $r = r_E$  of the macrolens, assuming that the velocity dispersion of the parent halo is  $\sigma \sim 200 \text{ km s}^{-1}$ .

From the astrometric shifts of the multiple extended images perturbed locally by a subhalo with respect to an unperturbed macrolensed image, we can break the degeneracy between the subhalo mass and the distance in the line of sight to the images if resolved at scale of an Einstein radius of the perturber [17, 19]. This is of great importance because otherwise we cannot determine whether the flux ratio anomaly is caused by more massive intergalactic halos in the line of sight or by less massive subhalos within the macrolens halo. Even if the density profile of the perturber is shallower than an SIS, we can make a distinction between models with different density profiles from astrometric shifts of the surface brightness profile within the source [19]. A direct detection of less massive  $10^3 M_\odot \lesssim M \lesssim 10^9 M_\odot$  subhalos within the parent halo will give a stringent constraint on the superWIMP and the WDM scenarios, which will be discussed in the next section.

### 3 Implications to SuperWIMP and WDM Scenarios

There exist many well motivated models for superWIMPs and WDM from particle physics. The natural candidates for the superWIMPs are gravitino and axino, that are superpartners of graviton and axion, respectively [8]. The right-handed sneutrino is also the candidate when neutrino masses are Dirac-type [21]. Others are Kaluza-Klein graviton and axion states in the UED [8]. As for the WDM, a light gravitino and sterile neutrinos have been discussed as such candidates. Thus, it is interesting to study the feasibility of probing or constraining those models in the near future experiments.

In the following, we refer “superWIMPs” as the particles whose interactions are weaker than the weak interaction, such as the gravitino which couples to other particles only gravitationally. SuperWIMPs are also assumed to be produced by the decays of heavier particles whose interactions are weak (*e.g.* WIMPs)<sup>#1</sup>.

In the CDM-based structure formation models, the structure of the universe forms in a hierarchical manner. Protohalos, which are the first virialized objects, appear first

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<sup>#1</sup>Usually conventional WDM particles do not satisfy both of these properties. In this sense, superWIMPs can be distinguished from conventional WDM particles.

after the mass density fluctuation becomes nonlinear, and larger objects form successively via their merger. In the ordinary WIMP models, the comoving damping scale of the power spectrum is typically  $(0.01 - 10)\text{pc}$ , depending on the Hubble radius at the kinetic decoupling temperature [22], and the protohalo mass is  $(10^{-12} - 10^{-4})M_\odot$  [23].

In the superWIMP or WDM scenarios, the primordial fluctuations on even larger scales can be damped because they have large velocity dispersion at the decoupling. Therefore, the protohalos become massive, and the total amount of subhalo mass inside the parent halo can be significantly reduced.

When the comoving damping scale of the DM power spectrum is  $R_{\text{cut}}$ , the protohalo mass is roughly given by

$$M_{\text{cut}} \simeq 1 \times 10^{11} M_\odot \left( \frac{R_{\text{cut}}}{1\text{Mpc}} \right)^3. \quad (3.3)$$

Here, the present matter density  $\Omega_{m,0} = 0.24$  and the Hubble constant  $h = 0.73$  are assumed [1]. In the previous section, we showed that the sensitivity of direct detection of subhalos inside the parent halo with submillilensing may reach intermediate mass  $\sim 10^3 M_\odot$  scales. Thus, when  $R_{\text{cut}} \gtrsim 2 \text{ kpc}$ , the future submillilensing experiments may directly detect such protohalos.

First, we discuss the damping scale in the superWIMP scenario and compare it with the sensitivities of future submillilensing. In the scenario, some WIMPs freeze out from the thermal equilibrium as the usual WIMP DM models, and superWIMPs are non-thermally produced from the WIMP decay, since the superWIMPs interact superweakly with the thermal bath. The scenario retains the property of the ordinary WIMPs that the observed relic density is naturally achieved. Furthermore, they are produced by decay of some long-lived massive particle  $X$ . Since they can have large velocities at the production epoch, the free-streaming damps the small-scale inhomogeneities. The superWIMP scenario is a possible explanation for the small-scale structure [24, 25].

The comoving damping scale is typically given by the free-streaming length of the superWIMP,  $R_{\text{fs}}$ , at the matter-radiation equality  $t_{\text{eq}}$ . When the long-lived particle has lifetime  $\tau_X$  and the superWIMP is produced with three-momentum normalized by its mass  $u(=p/m)$ , the comoving free-streaming scale  $R_{\text{fs}}$  is given by

$$R_{\text{fs}} = \frac{2t_{\text{eq}}}{a(t_{\text{eq}})} u_{\text{eq}} \left[ \log \left( \frac{1}{u_{\text{eq}}} + \sqrt{1 + \frac{1}{u_{\text{eq}}^2}} \right) - \log \left( \frac{1}{u} + \sqrt{1 + \frac{1}{u}} \right) \right] \quad (3.4)$$

where  $u_{\text{eq}} = (a(\tau_X)/a(t_{\text{eq}}))u$  and  $a(t)$  is the scale factor as a function of  $t$ .

When the superWIMPs are produced from decay of electrically-charged particles<sup>#2</sup>, the small-scale power may be further suppressed [28]. The charged particles are coupled

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<sup>#2</sup>It is recently pointed out in Ref. [26] that long-lived charged particles ( $\tau_X \gtrsim 10^3 \text{sec}$ ) lead to overproduction of  $^6\text{Li}$  in the Big Bang Nucleosynthesis due to the catalytic enhancement of the production. On the other hand, it is argued in Ref. [27] that there are a lot of ambiguities in the derivation.

to the photon-baryon fluid which oscillates on subhorizon scale. If the scale in question enters the horizon before the decay, the density fluctuation of such DMs cannot grow but oscillates, which gives a rise to the suppression on small scales. The damping scale is typically given by the Hubble radius at the decay time,  $H^{-1}(\tau_X)$ ,

$$R_{\text{ch}} = \frac{1}{a(\tau_X)} H^{-1}(\tau_X). \quad (3.5)$$

In Fig. 1 we show the damping scales  $R_{\text{cut}}$  of the charged and neutral  $X$  cases as functions of  $\tau_X$  and  $u$ . When  $X$  is neutral, the damping scale is determined by the free streaming scale  $R_{\text{fs}}$ . Longer lifetime  $\tau_X$  implies a larger free streaming scale, since the velocity dispersion at the radiation-matter equality time, which is proportional to  $a(\tau_X)/a(t_{\text{eq}})$ , becomes larger. When  $X$  is charged, the damping scale  $R_{\text{cut}}$  corresponds to the larger one of  $R_{\text{fs}}$  and  $R_{\text{ch}}$ . For  $u \ll 1$ ,  $R_{\text{cut}}$  is given by  $R_{\text{ch}}$ , since  $R_{\text{fs}}$  is  $\sim u \times R_{\text{ch}}$ . The hatched region with damping scales larger than  $\sim 1$  Mpc is constrained from Lyman alpha clouds [29]. The future submillilensing experiments may cover regions above the bold lines, which correspond to  $R_{\text{cut}} \gtrsim 2$  kpc. When the  $X$  lifetime is longer than  $\sim 1$  sec and the superWIMPs are produced with relativistic momentum ( $u \gtrsim 1$ ), the damping scale is larger than  $\sim 1$  kpc, which may be constrained by future observation if the subhalos with  $M \sim 10^3 M_\odot$  were discovered. When  $X$  is charged, the region with  $\tau_X \gtrsim 400$  sec may be also covered even in the small  $u$  cases.

Following Ref. [24], we indicated the parameter region which is suitable to solve the discrepancies in small-scale structure in Fig. 1. The region with  $R_{\text{cut}} \simeq (0.4 - 1.0)$  Mpc is suitable to solve the “missing satellite problem”, while the gray region with  $1 \lesssim u^{-3}(\tau_X/10^6 \text{sec})^{3/2} \lesssim 4$  is favored to solve the “cusp problem”, which requires a large DM velocity dispersion [30]. As we can see in Fig. 1, the model parameters corresponding to these regions can be well constrained by the future submillilensing observation.

For the illustrative purpose, we consider a superWIMP gravitino model, in which the gravitinos are produced by slepton or sneutrino decay. Sleptons or sneutrinos are assumed to be the lightest SUSY particles in the SUSY SM<sup>#3</sup>. The slepton and sneutrino lifetimes are given by [32]

$$\tau_X = 48\pi M_\star^2 \frac{m^2}{m_X^5} \left(1 - \frac{m^2}{m_X^2}\right)^{-1} \quad (3.6)$$

where  $M_\star = 2.4 \times 10^{18}$  GeV,  $m$  is the gravitino mass, and  $m_X$  is the mass of the parent WIMP  $X$ . In Fig. 2,  $R_{\text{fs}}$  (solid lines) is shown as a function of  $m$  and the mass difference between the gravitino and the parent particle ( $\Delta m = m_X - m$ ). Dashed lines indicate the damping scale for which  $R_{\text{ch}} > R_{\text{fs}}$ . The gray region which corresponds to explain the

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<sup>#3</sup>The lightest neutralino is also one of the candidates for the lightest SUSY particle in the SUSY SM. However, the decay produces hadronic shower to spoil the BBN if it is not photino-like. The long-lived slepton and sneutrino are not strongly constrained by the BBN, since their hadronic branching ratios are small. See Ref. [31] for the constraints on the superWIMP gravitino model.

“cusp problem” [24] may be excluded when the future submillilensing experiments find subhalos with mass smaller than  $\sim 1 \times 10^{(7-8)} M_\odot$ . It is known that the lifetime and the mass of the long-lived particles whose decay produces the superWIMPs, are constrained by the Big Bang Nucleosynthesis (BBN) and by the CMB Planckian spectrum, depending on the decay channels. When the hadronic modes are dominant in the decay, the BBN constrains the lifetime to be shorter than  $\sim 1$  sec [33]. Even if the hadronic shower is suppressed, the electromagnetic energy injection from the long-lived particle decay to the thermal bath is also constrained from the BBN while the constraint is weaker. In addition to it, the CMB Planckian spectrum also gives a constraint when the lifetime is longer than  $\sim 10^6$  sec [32].

The future submillilensing experiments are complementary to those constraints, since the damping scale  $R_{\text{cut}}$  is independent of the decay channels. For example, when the superWIMP gravitinos are produced from sneutrino decay, the energy injection to thermal bath is very tiny so that the constraints from the BBN and the CMB Planckian spectrum are very weak. Even in such a case, the submillilensing will still constrain the model.

Next, we discuss the damping scale in the WDM models. Among many WDM candidates discussed so far, the light gravitino with mass of order from  $10^{-6}$  eV up to 1 keV, which is likely to be the lightest SUSY particle in gauge-mediated models of supersymmetry breaking, is one of the well-motivated models from the viewpoint of particle physics [34, 35]. Such gravitinos are in thermal equilibrium at early times but decouples when the degrees of freedom  $g_*(T_D)$  is  $\mathcal{O}(100)$  where  $T_D$  is the decoupling temperature. Because the decoupling temperature of such species is higher than that of (active) massive neutrinos which play roles of hot dark matter, their velocity dispersion is not so large compared to that of massive neutrinos but non-negligible at the time of structure formation. Thus, they can act as the WDM. The comoving damping scale for the free-streaming for such gravitinos or any other thermal relic can be written as

$$R_{\text{fs}} \sim 0.84 \text{Mpc} \left( \frac{g_*(T_D)}{10.75} \right)^{1/3} \left( \frac{1 \text{keV}}{m_{\text{WDM}}} \right) \left( \frac{\langle p/T \rangle}{3.15} \right), \quad (3.7)$$

where  $\langle p/T \rangle$  is the mean momentum over the temperature. For thermally decoupled species, this factor gives almost unity, *i.e.*,  $\langle p/T \rangle / 3.15 \sim 1$ . The requirement from Lyman alpha clouds,  $R_{\text{fs}} \lesssim 1 \text{Mpc}$ , implies  $m_{\text{WDM}} \gtrsim 1 \text{keV}$ . On the other hand, the energy density of WDM can be written as  $\Omega_{\text{DM}} h^2 = (m_{\text{WDM}} / 94 \text{eV}) (10.75 / g_*(T_D))$ . Assuming  $\Omega_{\text{DM}} h^2 \sim 0.10$ , the mass of WDM should be  $m_{\text{WDM}} \sim 0.1 \text{ keV}$  even for  $g_*(T_D) \sim 100$ . We need to introduce more extra degree of freedom around  $T_D$  as  $g_*(T_D) \sim \mathcal{O}(10^3)$ . When the constraint on  $R_{\text{fs}}$  is improved to be  $\lesssim 1 \text{kpc}$ , the lower bound for the mass can reach  $m_{\text{WDM}} \sim 1 \text{ MeV}$  for  $g_*(T_D) \sim 100$ . Thus, the WDM scenario may face further difficulties if future submillilensing experiments would find small-mass subhalos.

Another well-motivated candidate for WDM is the sterile neutrinos [36]. Because they directly couple to the active neutrinos alone, they can be produced via neutrino oscillation. Although the evaluation of their energy density requires a numerical integration of the Boltzmann equation, some useful fitting formulae are available. The present energy density

of sterile neutrinos can be written as [37]

$$\Omega_{\nu_s} h^2 \sim 0.3 \left( \frac{\sin^2 2\theta}{10^{-10}} \right) \left( \frac{m_s}{100 \text{keV}} \right)^2, \quad (3.8)$$

where  $\theta$  is the mixing angle between the active neutrinos and sterile neutrinos and  $m_s$  is the mass of sterile neutrinos. The temperature at the time when the production is most efficient is [37, 38]

$$T_{\text{peak}} \sim 130 \text{MeV} \left( \frac{m_s}{3 \text{keV}} \right)^{1/3}. \quad (3.9)$$

The sterile neutrinos can damp the small-scale inhomogeneities via the free streaming in the same manner as the thermally decoupled WDM particles do. The free-streaming scale for a sterile neutrino WDM can also be obtained using Eq. (3.7) with different values for  $\langle p/T \rangle$  from that of the thermally decoupled ones. Because the sterile neutrinos are not in the thermal bath at early times, their distribution function deviates from that of a thermal one and the above factor can be  $\langle p/T \rangle / 3.15 \sim 0.9$  for the standard production mechanism [39]. Although there are some differences between the thermally decoupled WDM and the sterile neutrino WDM, they give the same predictions for the damping of matter power spectrum by identifying their masses as [40, 41]

$$m_s = 4.71 \text{keV} \left( \frac{m_{\text{thermal}}}{1 \text{keV}} \right)^{4/3} \left( \frac{0.10}{\Omega_{\text{DM}} h^2} \right)^{1/3}, \quad (3.10)$$

where  $m_{\text{thermal}}$  is the mass of thermally decoupled relics such as a light gravitino, which is denoted as  $m_{\text{WDM}}$  in Eq. (3.7). Because the shape of matter power spectrum is determined by the ratio  $m/T$  and the density parameter  $\Omega_{\text{DM}} h^2$ . Accordingly, the constraint on the sterile neutrino mass bound would be different from that on the thermally decoupled WDM mass bound<sup>#4</sup>. Using the above formula and assuming the damping scale as  $R_{\text{cut}} \lesssim 1 \text{ kpc}$  which can be reached by future submillilensing experiments, we can expect that the mass of sterile neutrinos can be constrained to be  $m_s \gtrsim 40 \text{ MeV}$ , which will be in conflict even with the current constraint  $m_s \lesssim 10 \text{ keV}$  [39, 42, 43]. Thus, we can obtain much more insight on the robustness of these scenarios from the future submillilensing experiments, as well as for the superWIMP scenarios.

Here some comments on the mixed dark matter scenario are in order. It is possible that, for example, gravitinos are produced not only from the decay of the next-lightest supersymmetric particles (NLSP) but also from thermal plasma. In this case, the present DM is composed of CDM and superWIMP. In such mixed DM scenarios, the damping of the small-scale structure is less significant in comparison with the models in which the superWIMPs make up all of the DM. To what extent the amplitude at small scales can be reduced depends on the ratio of the energy density of CDM and superWIMP DM. Detailed discussion on this issue is beyond the scope of this letter. Some discussions on

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<sup>#4</sup>The properties of the sterile neutrinos can also be constrained from the measurement of the X-ray flux, since they can contribute to the X-ray flux due to radiative decay [42]. See also Refs. [39, 43].



the matter power spectrum in such mixed models can be found in Refs. [44] and [45]. [44] analyzed models in which the superWIMPs are produced from the charged particle decay while [45] considered models in which the superWIMPs are produced from the neutral particle decay.

## 4 Conclusions and Discussion

We have shown that future observation of multiply-imaged QSO-galaxy lens systems with a high angular resolution  $\sim 0.01$  mas will prove the small-scale clustering properties of the DM halos down to  $\sim 10^3 M_\odot$ . The presence of  $\sim 10^3 M_\odot$  subhalos implies the comoving damping scale of the primordial fluctuations  $\sim 2$  kpc assuming that the observed subhalos retain the original mass during the merger process. The superWIMP and the warm DM scenarios that predict a larger damping scale (0.002–1) Mpc in comparison with (0.01–10) pc in the ordinary WIMP scenarios would be strongly constrained if the presence of such subhalos were proved.

In the superWIMP scenario, the superWIMP DM is produced from the long-lived particle  $X$  decay, and free-streaming of the superWIMPs damps the small-scale inhomogeneities. When the lifetime of  $X$  is longer than  $\sim 1$  sec, the damping scale is larger than  $\sim 1$  kpc unless the superWIMP and  $X$  masses are degenerate and the superWIMPs are non-relativistic at the production epoch. In addition, when  $X$  is a charged particle, it is coupled to the oscillating photon-baryon fluid before the decay. Therefore, the small-scale inhomogeneities inside the sound horizon cannot grow. The damping scale becomes larger than  $\sim 1 \text{ kpc} \times (\tau_X/100 \text{ sec})^{1/2}$  in the case of charged  $X$ . One of the natural superWIMP candidates is the gravitino produced from the slepton or sneutrino. In this case, the damping scale is typically larger than  $10^2$  kpc. The future submillilensing experiments, which cover the DM subhalos with mass  $\gtrsim 10^3 M_\odot$ , are important tests for probing such superWIMP scenarios. For the WDM scenarios, such as the light gravitino and the sterile neutrinos, the WDM mass would be further constrained.

The survivability of the protohalos during the merger process is under debate now. In the ordinary WIMP scenarios, it is claimed that most of the earth-mass protohalos are stable against the tidal stripping [46]. It is also discussed whether those halos are disrupted by interaction with stars [47]. On the one hand, subhalos that cross the galactic disk nearly perpendicularly or that fall off to the center of the parent galaxy are strongly disrupted by the tidal force, leading to a significant decrease in the total mass. On the other hand, subhalos that orbit on the plane nearly parallel to the disk or that reside in the low density region survive more-or-less intact.

For simplicity, we have assumed that the observed mass scale of the subhalo is equivalent to the lowerbound on the protohalo mass scale. In practice, however, the observable mass using gravitational lensing is limited to the one within a certain radius centered at the line of sight. As a result, there remains a certain ambiguity in estimating the mass scale of the observed subhalo. Observation of astrometric shifts of lensed QSO images

with a substructure in the surface brightness may help to reconstruct the subhalo mass density profile, thereby reducing the ambiguity [10].

It has been argued that the superWIMP and WDM scenarios can resolve the small-scale  $\lesssim 1$  Mpc discrepancies, such as the “missing satellite problem” and the “cusp problem” if the damping scale is as large as  $(0.4 - 1.0)$  Mpc. Therefore, future submillilensing experiments will shed a new light on these small-scale structure problems once the above ambiguities are removed.

## Acknowledgment:

We would like to thank Masashi Chiba for useful comments. This work is supported in part by the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan (No. 18034002 and 15540255 for JH).

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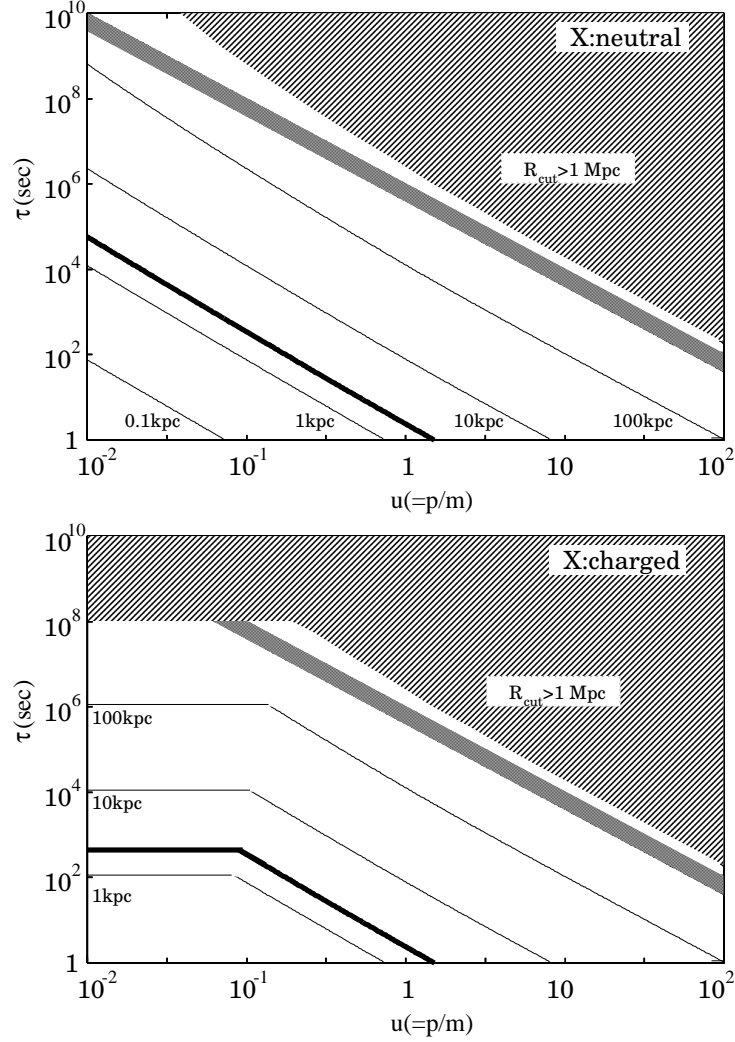


Figure 1: Damping scales of the DM power spectrum as functions of  $\tau_X$  and  $u$  in the superWIMP scenarios. The upper figure is for the neutral  $X$  case, and the lower one is for the charged  $X$  one. The (hatched) region with damping scale larger than 1 Mpc is constrained from Lyman alpha clouds. The gray region is favored to solve the cusp problem. The future submillilensing observations may cover the region above the bold lines which correspond to  $R_{\text{cut}} \gtrsim 2$  kpc.

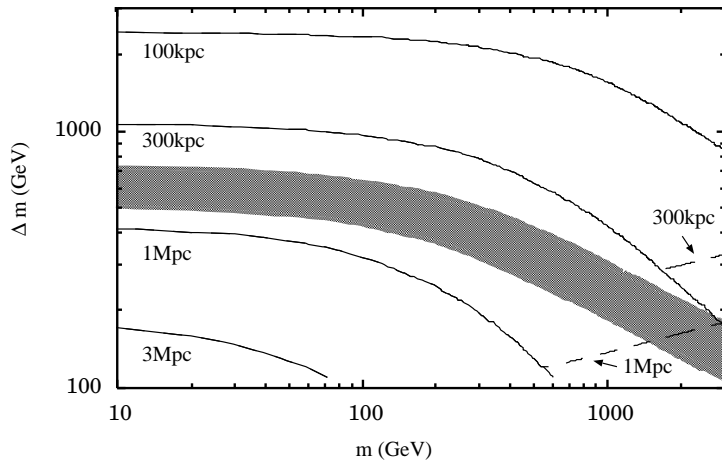


Figure 2: Damping scales for the gravitino superWIMPs produced by slepton or sneutrino decay.  $R_{fs}$ (solid lines),  $R_{ch}$ (dashed lines) are plotted as functions of  $m$  and  $\Delta m = m_X - m$ , respectively.  $R_{ch}$  is shown only for the case  $R_{ch} > R_{fs}$ . The gray region is favored to solve the cusp problem.